SOUND ABSORPTION IN A SHOCK WAVE

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It was shown in [1-4] that the reflection of a sound wave or its transmission through a shock front should be accompanied by attenuation or intensification of the wave is regarded as a discontinuity. In accordance with current representations [5, 6], a shock wave includes a viscous shock and a lengthy relaxation zone. Equilibrium is established with respect to translational and rotational degrees of freedom in the viscous shock and with respect to internal degrees of freedom in the relaxation zone. The result of the interaction of the shock and sound waves is determined by the relationship between the length of the sound wave and the width of the shock wave.

For low-frequency sound satisfying the condition $\omega\tau \ll 1$, where ω is the frequency of the sound and τ is the longest relaxation time, the viscous shock and the relaxation zone can be regarded as a discontinuity. In this case, the result of the interaction is found, for example, from the compatibility conditions. This was done in [7]. However, in contrast to [7], where investigators examined shock waves in a gas with a constant adiabatic exponent γ, the value of γ for strong shock waves in a polyatomic gas may be different on both sides of the wave because almost no vibrations are excited in front of the wave at the typical temprature $T \sim 300°K$ and the gas behind the wave is excited to oscillate.

The counter interaction of a shock wave and sound wave reinforces the latter. Meanwhile, with allowance for the difference in γ, the transmission factor, determined as the ratio of the amplitudes of the transmitted and incident sound waves, is equal to

$$K_t = \frac{M+1}{M_2+1}\frac{MM_2^2(\gamma_2-1)(\alpha-1)+(M+1)\alpha}{M_2^2(\gamma_2-1)(\alpha-1)+(M_2+1)\alpha}. \quad (1)$$

Here, M_2 is the Mach number in the flow behind the wave; $\alpha = \rho_1/\rho_2$; ρ is the density; the indices 1 and 2 denote parameters of the gas in front of the wave and at the end of the relaxation zone, respectively. For sufficiently strong shock waves, the enthalpy in the relaxation zone can be considered constant [6]. In this case

$$(7/2)kT + \varepsilon_v = (7/2)kT', \quad (2)$$

where k is the Boltzmann constant; ε_V is the mean vibrational energy per molecule; the prime denotes parameters of the gas at the beginning of the relaxation zone, while

$$M_2^2 = \frac{7\gamma_1/\gamma_2}{7+2(\varepsilon_v)_2/T_2}\frac{(\gamma_1-1)M^2+2}{2\gamma_1 M^2-\gamma_1+1}, \quad \alpha = \frac{7}{7+2(\varepsilon_v)_2/T_2}\frac{(\gamma_1-1)M^2+2}{(\gamma_1+1)M^2}.$$

The pattern is similar when a gaining sound wave is reflected from a shock wave [7], the only difference being that vibrational degrees of freedom should be considered in γ_2. The reflection coefficient, determined as the ratio of the amplitudes of the reflected and incident sound waves,

$$K_r = \frac{(M^2-1)^2}{j_0^2}\frac{2(2-\gamma_2)M^2-\gamma_2+1}{2\gamma_2 M^2-\gamma_2+1}, \quad (3)$$

where

$$j_0 = 1 + M^2\left[1+2\left(\frac{(\gamma_2-1)M^2+2}{2\gamma_2 M^2-\gamma_2+1}\right)^{1/2}\right].$$

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For nitrogen at T = 300°K, p = $9.8 \cdot 10^3$ Pa, and M = 7, we obtain $K_t = 32$ and $K_r = 0.18$.

For high-frequency sound at $\omega\tau >> 1$, $\omega\tau_0 << 1$ (τ_0 is the transit time), the pattern of interaction is more complicated. As before, the viscous shock can be considered a discontinuity and we can use Eqs. (1) and (3) to calculate sound reflection and transmission. However, when the sound propagates into the relaxation zone, it is now necessary to consider anomalous absorption. This absorption will be determined by the specific structure of the relaxation zone.

We will examine the absorption of sound in a zone of vibrational relaxation behind a shock wave front. Analysis of standard hydrodynamic equations in a Eulerian approximation together with the relaxational equation for the vibrational energy of a diatomic gas in the case of small perturbations of the form

$$a = a^0 + a_* \exp(ik_x x - i\omega t)$$

gives the following for the absorption coefficient ($\omega\tau >> 1$) [8]

$$\operatorname{Im} k_x = c^{-1/2}(c+1)^{-3/2} \frac{(c+q)(\varepsilon_{v0} - \varepsilon_v)/(kT) + c_V}{(kT/m)^{1/2}\tau}. \tag{4}$$

Here, c is the heat capacity associated with the translational-rotational degrees of freedom ($d\varepsilon = kcdT$); ε is the mean translational-rotational energy per molecule; ε_{V0} is the equilibrium value of vibrational energy with a given translational temperature T; c_V is the heat capacity associated with the vibrational degrees of freedom for the given translational temperature T ($d\varepsilon_{v0} = kc_V dT$); $q = BT^{-1/3}/3$, where B is determined from the Landau–Teller relation for the time of vibrational relaxation $\tau \sim (1/p) \cdot \exp(BT^{-1/3})$ [9]. It is evident from (4) that the presence of a nonequilibrium distribution of vibrational energy may lead to a substantial increase in the absorption coefficient. However, absorption of the type (4) in the relaxation zone needs to be taken into account only when it is greater than or comparable to Stokes absorption. If we use the following expression for the Stokes absorption coefficient

$$(\operatorname{Im} k_x)_s = \frac{\omega^2}{2\rho v_s^3}\left[\frac{4}{3}\eta + \varkappa\left(\frac{1}{c} - \frac{1}{c_p}\right)\right], \tag{5}$$

where η is the shear viscosity coefficient; v_s is the sonic velocity; κ is the thermal conductivity; c_p is the isobaric specific heat of the gas, then we obtain

$$\frac{\operatorname{Im} k_x}{(\operatorname{Im} k_x)_s} \sim \frac{(c+q)(\varepsilon_{v0} - \varepsilon_v)/(kT) + c_V}{c^{1/2}(c+1)^{3/2}\omega^2\tau_0\tau}. \tag{6}$$

Here, it is assumed that $\eta \sim \rho v_s^2 \tau_0$, $\varkappa \sim c\rho v_s^2 \tau_0$, while $v_s^2 = \gamma kT/m$. Since $c \sim 1$ in (6), the given relaxation mechanism predominates when

$$\omega^2\tau_0\tau \ll (c+q)(\varepsilon_{v0} - \varepsilon_v)/(kT) + c_V. \tag{7}$$

Condition (7) is satisfied for strong shock waves, when $\varepsilon_{v0}/(kT) \sim c_V \sim 1$ with $\omega^2\tau_0\tau << 1$. Conversely, Stokes absorption is predominant when $\omega^2\tau_0\tau >> 1$ for weak shock waves, where $(c+q)(\varepsilon_{v0} - \varepsilon_v)/(kT) + c_V \ll 1$. With allowance for the relaxation equation for vibrational energy

$$d\varepsilon_v/dx = (\varepsilon_{v0} - \varepsilon_v)/(u\tau)$$

we can write

$$\frac{\varepsilon_{v0} - \varepsilon_v}{kT} = \frac{D\rho_1\tau}{k\rho T}\frac{d\varepsilon_v}{dx}, \tag{8}$$

where u is the velocity of the gas behind the shock wave ($u = D\rho_1/\rho$); D is the velocity of the wave. After insertion of (8) into (4) and the use of (2), we obtain

$$\operatorname{Im} k_x = c^{-1/2}(c+1)^{-3/2}\left[\frac{c_V}{(kT/m)^{1/2}\tau} - \frac{7}{2}\frac{D\rho_1}{\rho_2 T_2} k \frac{c+q}{(kT/m)^{1/2}}\right]. \tag{9}$$

The first term in the right side of (9) corresponds to relaxational absorption in the propagation of sound in an initially equilibrium gas with a temprature T, while the second term corresponds to the effect of vibrational nonequilibrium.

To determine the integral absorption, it is necessary to consider that Eqs. (4) and (8) are valid in a stationary gas. In the relaxation zone, the sound wave, moving away from the front, moves together with the gas flow. This means that we should integrate over the length $\ell^* = v_s t$ in a coordinate system which moves with the flow. Thus, in the frame of reference connected with the front $\ell = \ell^* + ut$,

$$\int_0^{l^*} \operatorname{Im} k_x(x^*)\, dx^* = \int_0^{l} \operatorname{Im} k_x(x)\, \frac{v_s}{v_s + u}\, dx. \tag{10}$$

Since $T^{-1/3}$ changes little in the relaxation zone, the integral (10) from the second term of the right side of (9) is equal to

$$\int \operatorname{Im} k_x dx = 0{,}8\,(2{,}5 + q) \ln \frac{[(7M^2 - 1)(M^2 + 5)]^{1/2} + M^2 + 5}{[(7M^2 - 1)(M^2 + 5)]^{1/2} + \dfrac{7(M^2 + 5)}{7 + 2(\varepsilon_v)_2/T_2}}. \tag{11}$$

In this case, the absorption coefficient depends on M only indirectly, through q, because q is determined by the temperature behind the shock wave and this temperature is a function of M.

The absorption coefficient determined by vibrational nonequilibrium is equal to about 0.85 for the above-examined numerical example. The contribution of the relaxational term obtained by integrating (10) in the first term of (9) is equal to about 0.92. Such a difference becomes understandable if we consider that the first term in the right side of (4) exceeds the second term by a factor of $(c + q)\varepsilon'_{v0}/(kc'_V T')$ at the beginning of the relaxation zone, this factor amounting to approximately three in the present example; this difference decreases during relaxation, and the integral (11) exceeds the corresponding integral from the relaxation term by a factor of roughly two.

In contrast to the above case, sound passes through the relaxation zone twice in the interaction of a shock wave with a high-frequency sound wave which catches up to it. Thus, the total reflection coefficient will be equal to the product $K_1K_2K_r$, where K_1 is determined by the integral (10); K_2 is determined by an integral analogous to (10) but with the velocity difference $v_s - u$ in the denominator, which considers the change in the direction of the sound propagation; K_r is determined by Eq. (3) with $\gamma = 7/5$. For the numerical example examined above, we obtain $K_r = 0.07$ at $\omega\tau \gg 1$.

If the time of rotational relaxation τ_R is greater than τ_0, then when $\omega\tau_R \gg 1$ and $\omega\tau_0 < 1$ it is necessary to consider sound absorption in the zone of rotational relaxation. The investigation proceeds in a manner fully analogous to the previous case. Assuming $\tau_R \sim$ const in the relaxation zone, we have

$$\operatorname{Im} k_x = \left[\frac{3(\varepsilon_{R0} - \varepsilon_R)}{2kT} + 1\right] \Big/ \left[(15/4)^{1/2} (kT/m)^{1/2} \tau_R\right]. \tag{12}$$

It is evident from (12) that nonequilibrium absorption at the beginning of the relaxation zone exceeds anomalous absorption by a factor of 1.5. For the above numerical example, $K_t = 30$, $K_r = 0.12$ at $\omega\tau_R \ll 1$ and $K_t = 16$, $K_r = 0.02$ at $\omega\tau_R \gg 1$.

An evaluation made for Stokes absorption in a manner fully analogous to the above shows that Stokes absorption exceeds anomalous absorption at the frequencies $\omega(\tau_0\tau_R)^{1/2} \gg 1$. It should be noted that Eq. (5) is valid for the frequencies $\omega\tau_0 \ll 1$.

To evaluate the absorption of hypersound at $\omega\tau_0 \gg 1$, the front of the shock wave cannot be considered a discontinuity, and we must take into account the molecular structure of the viscous shock. We will use the model equation

$$u df/dx = (f - f_0)/\tau_0,$$

where f(x) is a nonequilibrium distribution function in the shock front; $f_0(x)$ is a Maxwell temperature distribution with a temperature determined by the mean store of translational energy; x is the distance from the center of the shock front. In this case, the

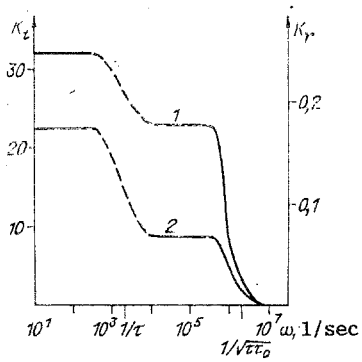

Fig. 1

problem reduces to the familiar problem of the propagation of hypersound in a gas with a Maxwellian distribution function f_0 [11]. The solution of this problem is well-known [12]:

$$v_s \approx 2(5kT/(3m))^{1/2}, \quad \mathrm{Im}k_x \approx 0{,}3\Omega(m/(kT))^{1/2}. \tag{13}$$

Here, Ω is the frequency in the coordinate system connected with the flow. To calculate the integral absorption coefficient, we will assume that f is a bimodal distribution function [13]. Then

$$\int \mathrm{Im}\, k_x dx \approx 2\omega \int \frac{(kT/m)^{1/2}}{(2\,(5kT/(3m))^{1/2} + D\rho_1/\rho)^2}\, dx, \tag{14}$$

where ρ and T are functions of x [13]; $\Omega(x) = \omega v_s/(v_s + u)$; ω is the frequency in the coordinate system connected with the front. For the given numerical example we obtain $K_t \sim \exp(-3\cdot 10^{-9}\omega)$. Since the integral temperature in the shock front increases exponentially, the integral absorption coefficient turns out to be the average of the values of absorption before and after the wave. The absorption coefficient for hypersound is very large (see (13)), and damping of the signal occurs over a length on the order of the mean free path.

Figure 1 shows results of calculation of the transmission coefficient K_t and reflection coefficient K_r (curves 1 and 2) for sound in the zone of vibrational relaxation. The pattern for rotational relaxation will be similar in the corresponding frequency range. It is evident from the figure that the above coefficients are both dependent on frequency. They have their largest value for low-frequency sound $\omega\tau \ll 1$. At frequencies $\omega\tau \sim 1$, the coefficients change due to the effect of vibrational relaxation (the form of the curve at $\omega\tau \sim 1$ was not examined and is indicated by dashes in the figure). In the frequency range $1/\tau \ll \omega \ll 1/(\tau\tau_0)^{1/2}$, the coefficients K_t and K_r are constant, since anomalous absorption (4) and the shock transmission coefficient (1) are independent of frequency. At frequencies $\omega \sim 1/(\tau\tau_0)^{1/2}$ and higher, the signal decays rapidly due to the predominance of Stokes absorption ($\ln K_{r,t} \sim -\omega^2$). The dependence of the coefficients K_t and K_r on frequency lead to a change in the form of the signal in the interaction of sound with a shock wave. In this case, a Doppler frequency shift will also occur. This phenomenon was examined in [1-4].

LITERATURE CITED

1. D. I. Blokhintsev, "Moving sound detector," Dokl. Akad. Nauk SSSR, 47, No. 1 (1945).
2. J. M. Burgers, "On transmission of sound waves through a shock wave," Proc. K. Ned. Akad. Wet., 49, 273 (1946).
3. V. M. Kontorovich, "Reflection and refraction of sound in shock waves," Akust. Zh., 5, No. 3 (1959).
4. Zh. S. Sislyan, "Interaction of perturbations with a shock wave in the unidimensional nonsteady motion of a gas," Zh. Prikl. Mekh. Tekh. Fiz., No. 3 (1963).
5. E. V. Stupochenko, S. A. Losev, and A. I. Osipov, Relaxation Processes in Shock Waves [in Russian], Nauka, Moscow (1965).
6. Ya. B. Zel'dovich and Yu. P. Raizer, Physics of Shock Waves and High-Temperature Gasdynamic Phenomena, Fizmatgiz, Moscow (1963).
7. F. V. Shugaev, Interaction of Shock Waves with Disturbances [in Russian], Izd. MGU, Moscow (1983).

8. A. I. Osipov and A. V. Uvarov, "Propagation of sound in a nonequilibrium gas," Vestn. Mosk. Univ. Fiz. Astron., 25, No. 6 (1984).
9. B. F. Gordiets, A. I. Osipov, and L. A. Shelepin, Kinetic Processes in Gases and Molecular Lasers [in Russian], Nauka, Moscow (1980).
10. L. D. Landau and E. M. Lifshitz, Continuum Mechanics, Gostekhizdat, Moscow (1953).
11. M. N. Kogan, Rarefied Gas Dynamics, Nauka, Moscow (1967).
12. L. Sirowich and J. K. Thurber, "Propagation of forced sound waves in rarefied gas-dynamics," J. Acoust. Soc. Am., 37, No. 2 (1965).
13. H. M. Mott-Smith, "The solution of the Boltzmann equation for a shock wave," Phys. Rev., 82, No. 6 (1951).

EXPERIMENTAL STUDY OF WAVE PROCESSES IN AN AQUEOUS SUSPENSION OF BENTONITE CLAY

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The practical value of studying wave propagation in water suspensions of bentonite stems from the broad use of these disperse systems in oil-well drilling.

As was shown in [1, 2], wave processes in bentonite have several characteristic features, including anomalous oscillation peaks in the incident wave which are considerably greater than the pressure of the initiating pulse, a successive increase in pressure in these peaks in a series of tests conducted with the same boundary conditions for initiation, etc. The studies [1, 2] described the results of investigations only in a dilute suspension of bentonite with a mass concentration c = 6% for the disperse phase in water. This concentration is near the critical concentration at which structure formation can take place in the mixture (see [3] and its bibliography, for example). Here, the impulsive pressures were recorded over a relatively short time interval — about 1 msec.

The goal of the present study is to further experimentally investigate waves in a system with a developed three-dimensional structure (c = 10%). The chosen time of observation of the waves is longer — on the order of 10 msec. This allows us to record the passage not only of the incident waves, but also of reflected waves, unloading waves, etc. along the entire tube.

1. Experimental Unit. The compression waves were initiated in a vertical shock tube [2]. The measurements were organized as follows. Three groups of pressure gauges were located along the low-pressure chamber (LPC) A, B, C at distances of 2, 5, 7 m from the diaphragm. The distance between two gages in a group $\Delta x = 0.25$ m. The triggering gauges controlled the measurement circuit — the front of the incident wave successively initiated electrical signals which alternately activated oscillographs and frequency meters operating in the slave rgeime. This was achieved by synchronizing the oscillograph readings with respect to time — the intervals measured by the frequency meters τ_1 and τ_2 corresponded to the time of passage of the wave from group to group or the time of delay of activation of the oscillographs relative to one another.

2. Trial Experiments. Water was chosen as the standard model liquid. Its low viscosity and the linear dependence of its volume on pressure up to 100 MPa made it possible to examine the results obtained from shock loading of the water column in an acoustic approximation.

A typical test is shown in Fig. 1. The intensity of the incident wave p_1 is close to the pressure of the driving gas in the high-pressure chamber (HPC) — $p_e = 2.4$ MPa. The initial pressure in the LPC $p_0 \approx 0.1$ MPa. The result is shown in the form of a kinematic curve in the coordinates x (height of the liquid column) — t (time) and illustrates the process of transmission of the compression wave (solid line) and rarefaction wave (dashed

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